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Theoretical evidence for negative Goos–Haenchen shifts

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Abstract. The Goos–Haenchen shift which occurs at total reflection from a glass–air interface is normally assumed to be positive, implying that the reflected beam of light is displaced with respect to the incident beam along the interface and in the sense and direction of the projection of the incident wavevector onto the interface. The aim of the paper is to show that the longitudinal displacement of electromagnetic radiation on reflection can be both positive and negative depending on the nature of the two media at whose interface reflection occurs. The shift is computed within the framework of classical electrodynamics by application of the principle of stationary phase. Two examples are worked out. The first is a case of a bundle of microwaves incident onto a corrugated metallic surface, whereas the second involves electromagnetic radiation falling on a magnetised plasma. In both examples the longitudinal displacement is found to be positive, negative or zero depending on the angle of incidence and other parameters.

It is also shown that negative Goos–Haenchen shifts do not contradict the law of conservation of photon momentum.

1. Introduction and general considerations

It has been demonstrated experimentally by Goos and Haenchen (1947) that electromagnetic radiation totally reflected from a glass–air interface suffers a longitudinal displacement in the plane of incidence before re-entry into the glass, as shown in figure 1(a). The history of the problem goes back to the turn of the century, and the various optical and microwave aspects have been reviewed in detail by Tamir (1972, 1973).

It has been shown by Fedorov (1955), Schilling (1965) and Imbert (1972) that apart from the longitudinal shift shown in figure 1 there is also a lateral displacement normal to the plane of incidence (the plane of the paper), which exists only if the

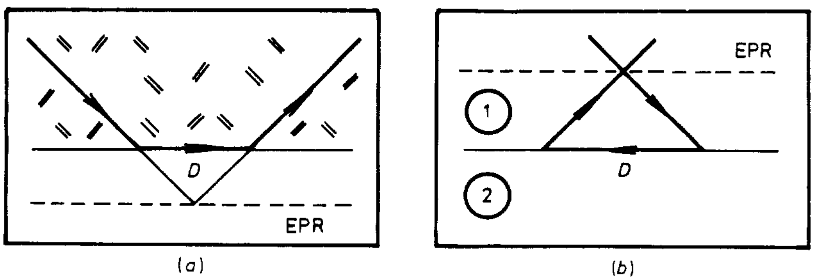

Figure 1. (a) The Goos–Haenchen experiment. (b) The ray picture of a negative shift. EPR stands for 'equivalent plane of reflection'.

polarisation of the incident wave is not one of the fundamental polarisations (s and p). The lateral shift was also observed in the microwave region by Cowan and Aničin (1974).

Both longitudinal and lateral shifts can be computed on classical electrodynamics by two concurrent procedures, one relying on plane wave spectra and the other on the Poynting theorem. Both methods yield longitudinal shifts of the order of several wavelengths of the radiation and lateral shifts smaller by an order of magnitude, but do not exactly agree. The discrepancy between the two procedures is probably attributable to a number of causes, such as the use of fields which are not solutions of Maxwell's equations, the fact that the distinction between lateral, surface and leaky waves is seldom embodied in the calculation and the neglect of the finite width of the irradiating beam. Regarding this last point, Horowitz and Tamir (1971) have conducted a complete analysis with a Gaussian distribution of field within the incident beam and proved that the displacement actually depends on the width of the primary beam. The analysis also shows that the principle of stationary phase, which is frequently invoked to deal with integrals resulting from the superposition of plane waves, yields reliable results for almost all angles of incidence for beams wide with respect to the wavelength, except in the vicinity of the critical angle. As the angle of incidence approaches the critical value, the displacement computed on the principle of stationary phase goes to infinity, whereas the finite-beam theory yields large but finite displacements.

The aim of this paper is to draw attention to the fact that the longitudinal displacement as computed by the application of the principle of stationary phase within the method of plane-wave spectra can be positive, negative and zero. Two examples of reactive surfaces yielding negative shifts have been found. The first is a corrugated metallic surface illuminated by a beam of microwaves, described in § 2. The second example is that of electromagnetic radiation incident from free space onto a semi-space of plasma, permeated by a magnetic field normal to the plane of incidence, which is described in § 3.

The general case of a negative longitudinal displacement is shown, using a ray picture, in figure 1(*b*). We see that the equivalent plane of reflection (EPR) is now above the actual interface between the two media. Power is carried from the incident beam to the reflected beam by some form of inhomogeneous wave, which now travels in the opposite sense to the similar wave in figure 1(*a*). A first reaction to the situation depicted in figure 1(*b*) is that it contradicts the law of conservation of momentum of the photon. Indeed, the component of momentum parallel to the interface is opposite for the inhomogeneous wave and the incident or reflected photons. The contradiction is probably only apparent. Applying Heisenberg's indeterminacy principle to the situation in figure 1(*b*), we have $\Delta p\, \Delta x \geqslant h$ and as $\Delta x \sim D$ and $D \sim \lambda$, $\Delta p \geqslant h/\lambda = p$. The indeterminacy in momentum is of the order of momentum itself; for the situation in figure 1(*b*) we only require that the indeterminacy be twice the momentum: $\Delta p = 2p$. The law of conservation therefore applies to the initial and final states of the photon; the inhomogeneous wave has to be regarded as a virtual state, akin to virtual energy states.

2. The corrugated surface

The surface shown in the inset of figure 2 was examined by Cullen (1954) as a guide

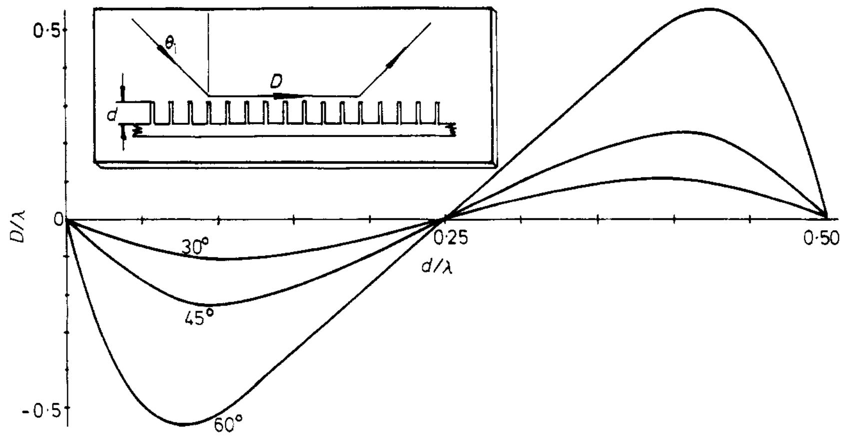

Figure 2. The corrugated surface. Base and corrugations are highly conductive. The period of the corrugations is much smaller than the wavelength. The longitudinal displacement versus corrugation depth d is shown in the diagram for three angles of incidence ($\theta_i = 30°$, 45° and 60°).

capable of carrying surface waves in the microwave region. It is not a diffraction grating, in fact the separation between the conductive corrugations is much smaller than the free-space wavelength. The reflection coefficient for a p polarised electromagnetic wave incident at θ_i to the surface normal is

$$R = \frac{Z_0 \cos\theta_i - iX}{Z_0 \cos\theta_i + iX} \tag{1}$$

where Z_0 is the impedance of free space, $X = Z_0 \tan(kd)$, d is the depth of the corrugations and k the free-space wavenumber. The principle of stationary phase yields generally for the displacement $D = k^{-1}\, d\phi/ds$, where ϕ is the phase of the reflection coefficient and $s = \sin\theta_i$. The resulting shift is given by

$$D = -\frac{2}{k}\,\frac{\tan(kd)\tan\theta_i}{\cos^2\theta_i + \tan^2(kd)}. \tag{2}$$

The displacement is negative if the depth of the corrugations is less than a quarter wavelength, but can be positive if $\tan(kd) < 0$.

A plot of the displacement D versus the depth of the corrugations d is shown in figure 2. The parameter in the figure is the angle of incidence.

3. Magnetised plasma

The application of the principle of stationary phase to the incidence of a bundle of electromagnetic radiation on a semi-space of magnetised plasma also leads to longitudinal displacements that can be either positive or negative depending on the circumstances. The geometry of the problem is shown in the inset of figure 3. The time-independent magnetic field B is normal to the plane of incidence, which contains the E vector of the wave (p polarisation). The displacement depends on the angle of incidence, the electron number density of the plasma and the magnetic induction B. The last two quantities appear in the theory in normalised form and are represented

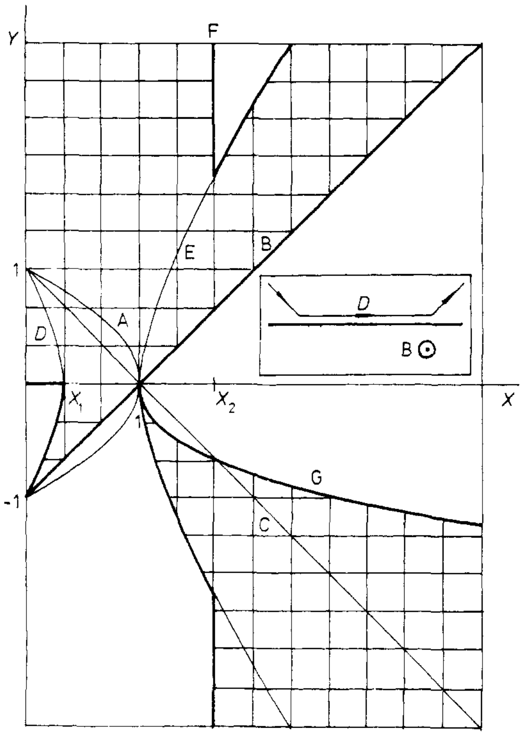

Figure 3. The geometry of an incident beam of electromagnetic radiation displaced by reflection from a magnetised plasma is shown in the inset. The variables X and Y are proportional to the electron number density and magnetic induction, respectively. The displacement is positive in the regions of the diagram which are covered by the mesh of squares and negative in the domains which are left blank. The various curves appearing in the diagram are defined in the text.

by the standard magneto-ionic notations X and Y, where X stands for the square of the ratio of plasma frequency and radiation frequency, whereas Y is the cyclotron frequency divided by the radiation frequency.

The reflection coefficient R of the transversally magnetised plasma is given by

$$R = \frac{n \cos \theta_i - \cos \theta_r - L \sin \theta_r}{n \cos \theta_i + \cos \theta_r + L \sin \theta_r}, \tag{3}$$

where θ_i and θ_r are the angles of incidence and refraction, respectively, n is the plasma refractive index given by

$$n^2 = \frac{(1-X)^2 - Y^2}{1 - X - Y^2} \tag{4}$$

and L is the coefficient of longitudinal polarisation of the plasma

$$L = \mathrm{i}L_i = \frac{\mathrm{i}XY}{1 - X - Y^2} \tag{5}$$

defined originally as the ratio of the longitudinal and transverse components of the electric field within an elementary plane wave in the plasma.

The phase of the reflection coefficient depends primarily on the sign of the square of the refractive index and two cases must be distinguished first: (*a*) $n^2>0$; (*b*) $n^2<0$. A second distinction to be made is whether $n^2>s^2$ or $n^2<s^2$ with $s=\sin\theta_i$.

The longitudinal displacement is obtained from the phase of the reflection coefficient via the relation $D=k^{-1}\,d\phi/ds$ and is given by

$$D=\frac{2n^2}{k[(1-s^2)(s^2-n^2)]^{1/2}}\,\frac{s(1-n^2)-L_i(s^2-n^2)^{1/2}}{n^4(1-s^2)+[(s^2-n^2)^{1/2}-L_is]^2} \tag{6}$$

if $n^2>0$, $n<s$. Similar expressions, valid for $n^2<0$ and $n>s$ were also obtained. The results of the analysis are summarised graphically in figure 3 which shows the regions in X–Y space where the displacement is positive (squares) and negative (blank).

The various curves that appear in figure 3 are as follows. Curve A is a parabola given by $n^2\to\pm\infty$, $Y=\pm(1-X)^{1/2}$ and it represents in fact the locus of those points in X–Y space where the radiation frequency equals the upper hybrid frequency of the magnetised plasma. B and C are two straight lines given by $Y=\pm(X-1)$ on which the refractive index of the plasma vanishes ($n=0$). All other curves in the diagram depend upon the angle of incidence, and the plot in figure 3 is in fact different for different angles of incidence. E and F are the two branches of the hyperbola $Y=\pm(X-1)^{1/2}(X-\cos^2\theta_i)^{1/2}/\cos\theta_i$ which is the locus of all points with $n^2=s^2=\sin^2\theta_i$. F–F is a straight line given by $X=X_2=1/\cos^2\theta_i$. Curve G is a part of the parabola $Y=-\tan\theta_i(X-1)^{1/2}$ at which the displacement is zero; the passage from positive to negative displacements here is continuous and occurs as a consequence of the variation of either the magnetic field, electron number density or angle of incidence.

With the E vector normal to the plane of incidence (s polarisation) the displacement exists only if the plasma is impenetrable for the radiation and is always positive.

As a check of the procedure the same formalism was also applied to the Goos–Haenchen experiment, which yielded the results obtained by Schilling (1965). The shift is positive in this case for both polarisations of the incident wave (s and p).

4. Conclusions

As far as the authors are aware the longitudinal Goos–Haenchen shift has always been considered positive. This paper serves to demonstrate that negative displacements do not contradict conservation of photon momentum and provides two examples of negative shifts obtained by *bona fide* application of the principle of stationary phase. It is also to be believed that negative shifts can be obtained by the energy theory, as the sign of the Poynting flux of the inhomogeneous wave depends upon the actual physical situation. As far as experimental verification is concerned, the corrugated surface seems more amenable to experiments than the magnetised plasma case.

Apart from the purely fundamental aspects of the Goos–Haenchen shift, the issue of negative displacements gains in practical importance with the development of integrated optics and the use of semiconductors in this field.

Note added in proof. Transverse and longitudinal shift experiments in the microwave region are now described in full detail in Cowan and Aničin (1977).

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